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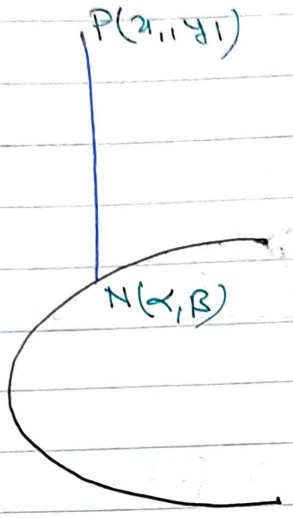
# THEOREM OF THREE NORMALS.

To prove that from any point $(x_1, y_1)$ 3 normals drawn to the parabola $y^2 = 4ax$.

Proof:




given eqn of parabola is

$$y^2 = 4ax \quad \text{———} \quad (1)$$

& the point is $P(x_1, y_1)$

From P we draw a Normal to the parabola (1) whose ~~fo~~ coordinate of foot ~~of~~ is $N(\alpha, \beta)$.

Since, $N(\alpha, \beta)$ lies on the parabola, so it satisfy its equation.

i.e.

$$\beta^2 = 4a\alpha. \quad \text{———} \quad (2)$$

The eqn of normal to the parabola (1) at the point $N(\alpha, \beta)$ is.

$$\frac{y-\beta}{\beta} = \frac{x-\alpha}{-2a}$$

$$\frac{x-\alpha}{2a} + \frac{y-\beta}{\beta} = 0 \quad \text{--- (3)}$$

It passes through $P(x_1, y_1)$ so it satisfy its equation:
i.e.

$$\frac{x_1-\alpha}{2a} + \frac{y_1-\beta}{\beta} = 0.$$

or, $$\frac{x_1\beta - \alpha\beta + 2ay_1 - 2a\beta}{2a\beta} = 0.$$

or, $x_1\beta - \alpha\beta + 2ay_1 - 2a\beta = 0$

$\beta^2 = 4a\alpha$
$\alpha = \frac{\beta^2}{4a}$

or, $x_1\beta - \frac{\beta^2}{4a}.\beta + 2ay_1 - 2a\beta = 0$

or, $4ax_1\beta - \beta^3 + 8a^2y_1 - 8a^2\beta = 0.$

or, $-\beta^3 + 4a(x_1 - 2a)\beta + 8a^2y_1 = 0$ --- (3)

This is cubic eq$^n$ in $\beta$, so it has 3 roots say. $\beta_1, \beta_2, \beta_3$
For these values of $\beta$ we get three values of $\alpha$.
This shows that we get three foot of Normal.
Hence any point 3 Normals drawn to the parabola.

Proved.